# A ROBUST TIMING AND FREQUENCY OFFSET ESTIMATION SCHEME FOR ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING (OFDM) SYSTEMS

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<u>Abstract</u> - This paper presents a novel, robust technique to estimate timing and frequency offset in an Orthogonal Frequency Division Multiplexing (OFDM) system without the use of pilot signals. We first present an overview of an OFDM system for high-speed wireless data communications [1], including the particular parameters chosen for our design. We then describe some prior attempts to estimate timing and frequency offset [2][3][4][5]. We finally present our approach and show simulation results in the presence of the variety of severe impairments that are likely to be observed in a typical outdoor PCS environment.

#### I. INTRODUCTION

The problem of data communications over a wide cellular-like coverage area, with highly mobile users sharing access to high-speed networks, is receiving increasing interest. OFDM is one technique that has been proposed to offer substantially higher data rates than those currently available to the mobile user. As with any communications system, effective, reliable, and efficient techniques are required that will allow synchronization of remote terminal. This paper presents a simple synchronization technique that provides robust offset estimates over a wide variety of channel conditions without the use of bandwidth-consuming pilots.

#### II. OFDM OVERVIEW

For the Advanced Cellular Internet Service (ACIS) we have been investigating [1], it is desirable to offer a peak user data rate of at least 384 kb/s in a high mobility wide-area cellular-like system coverage area. For a single carrier system, this would either entail using a very complex constellation with many bits per symbol or a very high symbol rate. Using a dense signalling constellation is undesirable for a wireless system: noise and amplitude variation (fading) make it difficult to reliably detect which constellation point was sent. Likewise, signalling at a high symbol rate is equally undesirable: the multipath nature of the wide area channel would require a complex high speed equalizer or similar technique to deal with the time dispersion of the transmitted signal.

For these reasons, OFDM is used to provide acceptable performance in the fading, multipath RF environment while promising high peak data rates. OFDM techniques have been used in high-speed wireless LANs, digital audio broadcast systems and wireline high-speed data communications systems.

The architecture of the ACIS prototype system is illustrated in Figure 1. Figure 2 shows some of the system parameters. As shown in the figures, 189 individual tones are QPSK modulated at 3.466 kbaud. This relatively low symbol rate mitigates the effect of the dispersive multipath channel. At the same time, 189 QSPK modulated tones can convey 378 bits of channel data per OFDM block or an aggregate bit rate of 1.31 Mb/s. A rate ½ Reed-Solomon coder plus frame and signalling overhead leaves enough channel capacity to readily support a 384 kb/s peak end user data rate.

The use of the Fast Fourier Transform (FFT) in the transmitter and receiver greatly simplifies the overall system structure. Processing each individual tone's modulation in the frequency domain reduces the modulation/demodulation process to one of setting and comparing a single complex number for each tone.

Figure 1 includes means to compensate for the inevitable frequency and timing offset that will be present in any practical communications systems. Figure 2 illustrates the time domain envelope of the OFDM slot but implicitly assumes correct time alignment. The rest of this paper will describe a

technique to accurately estimate the necessary timing and frequency adjustments to allow the receiver to correctly process the samples of the OFDM waveform.



Figure 1 - OFDM system architecture



# Figure 2 - OFDM waveform envelope and system parameters

One feature of the OFDM waveform portrayed in Figure 2 that should be noted is the cyclic extension. The cyclic extension is a copy of samples from the set of FFT samples with a fixed modulo offset. With the 512-point FFT illustrated, the first FFT sample would be copied into the 513th sample position, the second FFT sample to the 514th position, etc. The cyclic extension makes the signal more robust against time dispersion in the channel and timing offset in the receiver. Up to the interval of the cyclic extension, these timing offsets and "echoes" will be repetitions of the same samples, insuring that the amplitude of the received spectrum will be uniform

# III. PRIOR SYNCHRONIZATION TECHNIQUES

Several prior techniques [2][3][4][5] have dealt with the problem of estimating time and frequency offset, either jointly or individually, in OFDM systems. Many have proposed the use of pilot symbols or tones. While the use of pilot signals may make the synchronization problem easier, it reduces the overall system efficiency since signal power is used that could otherwise have been used for user data.

Some techniques [5] rely on the inherent redundancy in the OFDM time waveform, correlating parts of the waveform (i.e., the FFT samples) with other parts of the waveform (i.e., the cyclic extension samples). Others [3] rely on sets of pilot bursts or avoid multiple pilot bursts by creating redundancy in the time domain waveform itself[2].

Finally, previous papers [6][7] have analyzed the effects of these synchronization errors. In simple terms, frequency offset destroys the harmonic structure of the tones and thus the orthogonality. The effect of timing offset will become obvious as the proposed technique is described below.

## IV. PROPOSED APPROACH

#### **Timing offset**

First, consider the effect of timing offset on the OFDM tones. For a set of modulated tones,  $X_m$ , the sampled time domain waveform  $x_n$  is given by:

$$x_n = \frac{1}{\sqrt{N}} \cdot \sum_{m=0}^{N-1} X_m \cdot e^{\frac{j \cdot 2 \cdot \pi \cdot n \cdot m}{N}}$$

Ignoring the other channel impairments, the received waveform, y(t), is a time shifted version of the reconstructed transmit signal x(t):

$$y(t) = x(t - \Delta t)$$

By the time shifting property of the Fourier Transform, the sampled spectrum at the receiver is

$$Y(\omega) = X(\omega) \cdot e^{-j \cdot \omega \cdot \Delta t}$$

so the received signal's discrete spectrum is:

$$Y_m = X_m \cdot e^{\frac{j \cdot 2 \cdot \pi \cdot \Delta t}{N}}$$

Rearranging terms:

$$\Delta t = \frac{N}{2 \cdot \pi} \cdot \left( \arg(\frac{Y_i}{X_i}) - \arg(\frac{Y_{i+1}}{X_{i+1}}) \right)$$

Note that the  $X_i$  factors are the modulation on the transmitted tones. It will be shown later that this can be ignored. Also note that, while all the  $Y_i$ 's and  $Y_{i+1}$ 's are distinct, they are all related to each other by the same  $\Delta t$ . In other words, each tone pair provides a separate noisy estimate of the same  $\Delta t$ .

Figure 3 illustrates the uncorrupted transmit signal, plotting tone phase as a function of tone number for a 189 tone QPSK modulated transmit signal. Figure 4 shows the same signal at the receiver with a one sample timing offset.



Figure 3 - Transmit tone phase vs. tone number



Figure 4 - Receive tone phase with 1 sample offset

As shown, the phase rotation of individual tones directly corresponds to the timing offset. Differentially detecting the phase difference between adjacent tone pairs is the essence of the proposed scheme.



Figure 5 – Differential-in-frequency constellation

This "differential-in-frequency" detection creates a signal "constellation" as shown in Figure 5. The rotation of the constellation points is the manifestation of the timing offset, in this case 25 samples

Since the transmitted signal uses QPSK modulation, each pair of tones will differ from each other by a multiple of  $\pi/2$  in addition to the timing offset rotation. This is apparent in Figure 5 since there are four clusters of constellation points. If the transmitted data values are known, e.g., by using the demodulated and decoded data values, the individual points could be rotated by an appropriate multiple of  $\pi/2$ . Alternatively, by raising the individual points to the 4<sup>th</sup> power, the effect of QPSK modulation is removed and the signal constellation shown in Figure 6 results. The timing control signal is obtained by finding the centroid of the constellation. Alternatively, since all the points will lie on the positive real axis when the timing instant is correct, the individual vectors can be summed, ideally resulting in a straight line along the positive real axis. Any deviation from this is directly proportional to the amount of timing offset.



**Figure 6 - 4th power constellation** 

Frequency offset has no effect on the timing offset estimation. Frequency offset causes a uniform rotation of all tone phases (in addition to an inconsequential perturbation of amplitude due to inter-tone interference). Figure 7, shows that, while frequency offset modifies the timing offset constellation, it does not move the centroid of the clusters. The figure shows the effect of 25 samples of timing offset with a frequency offset of one quarter of the tone spacing.

Raising the signal to the 4<sup>th</sup> power to remove its modulation requires less signal processing complexity than attempting to estimate and correct for signal modulation, but also has its limitations. There is a maximum rotation of the constellation before a phase ambiguity is created. This is not a practical limitation, however, since the timing offset estimation scheme will linearly track an offset of 1/8<sup>th</sup> the FFT size



Figure 7 - Timing plus frequency offset

#### **Frequency offset**

Having addressed the issue of timing offset estimation, a similar technique can be used for estimating the frequency offset. First, consider the same transmit signal shown in Figure 3, this time with frequency offset. The received signal tone phases are illustrated in Figure 8.



Figure 8 - Receive tone phase with 1/4 tone offset

It can be seen that the tone phases are all shifted slightly. In addition, there is a slight variation in signal phase from tone to tone due to inter-tone interference. Examining the complex output of the FFT, the four distinct QPSK phases are evident, albeit with a slight rotation of the constellation as shown in Figure 9.



# Figure 9 – Complex FFT output with $\Delta f = 1/4$ of tone spacing

As above, the effect of modulation can be removed by raising the constellation points to the 4<sup>th</sup> power as shown in Figure 10. This, again, gives a set of phasors that should nominally lie on the positive real axis. As before, any deviation from the real axis translates directly into the amount of frequency offset present in the received signal.



Figure 10 - 4th power of complex FFT output

Note that due to interference between the nonorthogonal carriers, the amplitudes of these constellation points have been altered, not the centroid.

#### V. PERFORMANCE

Both the timing and frequency offset estimation algorithms were tested under a variety of channel conditions. For brevity, the performance of the timing estimation algorithm will be emphasized here. For all of these tests, the performance measure was how close to the correct amount of timing offset the algorithm indicated, relying on the information in a single OFDM block. Obviously, in a "tracking" mode with averaging across several blocks, performance would be expected to improve.

First, the estimation algorithm was tested with additive white gaussian noise to see if it could track timing at the limits of system operation. Figure 11 shows the timing constellation with a 6 dB SNR. Even under this low SNR condition, the timing offset estimate was within a fraction of a sample.



Figure 11 - Timing constellation with 6 dB SNR

As described above and illustrated in Figure 7, simultaneous timing and frequency offset were tested. Even with a frequency offset equal to the tone spacing, timing offset estimation was not degraded.

To understand the robustness of the offset estimation scheme, one must consider how impairments affect the estimation algorithm. If the average SNR on the channel is acceptable for data transmission, many of the tones will have a good SNR. If a tone pair has good SNR, differential comparison will provide a good phase estimate. Otherwise, if one or both tones have poor SNR, most likely the pair is subjected to frequency selective fading. This pair cannot be relied on to provide a good phase estimate. However, the nature of the phase detection insures that the amplitude of this phasor will be small, being product of the two tone amplitudes. Thus, there is inherent weighting of phase estimates with the most accurate contributing a maximum amount to the overall estimate while the least accurate contribute little.



#### Figure 12 - Timing adjustment vector. $\Delta f$ =1.2 tones, $\Delta t$ =45 samples

Figure 12 shows the cumulative adjustment vector. Even with large amounts of timing and frequency offset, it can be seen how a large set of individually noisy measurements combine to accurately track offset. All available tones were not needed. As few as 3 tones worked on a good channel, while 15-25 were generally sufficient. Thus, signal processing load, channel conditions, and performance tradeoffs are viable.

In an attempt to find the limits of the timing offset algorithm's robustness, the channel was degraded by inserting significant amounts of frequency selective fading. With very high levels of exponentially distributed delay distortion or even two equal rays, the estimation algorithm successfully tracked the centroid of the channel impulse response. Alternating tones were completely cancelled with 256 samples of delay between two equal rays. Stable timing estimates were obtained even here because the transformed spectrum has zero energy at alternate tones if the timing offset is zero. Perturbation in the timing offset generated a correction to adjust the timing instant appropriately.

## VI. CONCLUSIONS

We have presented a time and frequency offset estimation scheme for OFDM systems. The technique works reliably under a wide range of impairments including noise, frequency selective fading, time dispersion, etc.

The scheme presented allows robust timing offset estimation in the presence of large amounts of frequency offset and provides equally robust estimates of frequency offset. It can be used for initial coarse offset estimation as well as providing a direct proportional estimate for tracking. Inherent features of the frequency domain signal are used, without additional pilot signals. Complexity and performance can be readily traded off.

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